REGULARITY FOR SINGULAR RISK-NEUTRAL VALUATION EQUATIONS

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Abstract

The arbitrage-free price of a security can be computed as the solution of a Partial Integro-Differential Equation (PIDE), the general form of which is

$$\begin{cases} \partial_t u(t,x) + Lu(t,x) - c(x)u(t,x) = f(t,x), & (t,x) \in (0,T) \times D, \\ u(T,x) = \phi(x), & x \in D, \end{cases}$$

$$Lg(x) = \nabla g(x) b(x) + \frac{1}{2} \mathrm{tr}\left(\nabla^2 g(x) a(x)\right) + \int_D \left[g(z) - g(x)\right] m(x, dz).$$

where D is a (possibly unbounded) starshaped open subset of $\mathbb{R}^d$.
Often, these PIDEs have singular diffusion matrices and coefficients that are not Lipschitz up to the boundary. In this talk we present existence and uniqueness results of the viscosity solution to the valuation equation for a general jump-diffusion model with locally Lipschitz continuous coefficients.
Our assumptions allow the diffusion matrix $a = \sigma\sigma^\top$ to be singular and both σ and the drift b to loose Lipschitz continuity at the boundary of the state space D.
When our assumptions are verified, the underlying stochastic process does not reach the boundary of D, so that our formulation does not require any boundary conditions to be specified.

The issue of regularity of the viscosity solution is also investigated. When L is a purely differental operator, regularity results are well known. When L is an integro-differential operator, regularity results are available in the case when the second order term of L is uniformly strictly elliptic and under some assumptions on m (see e.g. Garroni and Menaldi (2002)).
In contrast, here we are specifically interested in the case of a degenerate σ, i.e. when $\sigma(x)p = 0$ for some $p \in \mathbb{R}^d$, at some point $x \in D$, or possibly for all $x \in D$ (*structural degeneracy*).

We discuss the degree of smoothness of the solution for a class of strongly degenerate integro-differential operators L.

Our results apply to Asian option pricing (in stochastic volatility models as well), to path-dependent volatility models and to jump-diffusion stochastic volatility models.

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