Monotonicity formulas for operators with variable coefficients

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We will discuss the recent extension of the results of Caffarelli-Jerison-Kenig [Ann. of Math. (2) **155** (2002)] and Caffarelli-Kenig [Amer. J. Math. **120** (1998)] by establishing an almost monotonicity estimate for pairs of continuous functions satisfying

$$u_\pm \geq 0, \quad \mathscr{L} u_\pm \geq -1, \quad u_+ \cdot u_- = 0$$

in an infinite strip (global version) or a finite parabolic cylinder (localized version), where $\mathscr{L}$ is a uniformly parabolic operator

$$\mathscr{L} u = \mathscr{L}_{\mathscr{A},b,c} u := \operatorname{div}(\mathscr{A}(x,s)\nabla u) + b(x,s) \cdot \nabla u + c(x,s)u - \partial_s u$$

with double Dini continuous $\mathscr{A}$ and uniformly bounded b and c. We also prove the elliptic counterpart of this estimate.

This closes the gap between the known conditions in the literature (both in the elliptic and parabolic case) imposed on $u_\pm$ in order to obtain an almost monotonicity estimate.

As a demonstration of its use, we prove the optimal $C^{1,1}$ regularity in a fairly general class of quasilinear obstacle-type free boundary problems.